CONVECTIVE HEAT-TRANSFER EQUATION IN THE ENTRANCE SECTION OF AN ANNULAR CHANNEL

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The authors present results of experimental investigations of convective heat transfer in annular drying channels, and examine their possible practical use.

For cooling various items of cylindrical shape one ordinarily uses annular channels formed by the surface of the item being cooled and a tube surrounding it, and with gas or liquid pumped along the channel. It is known that in the initial thermal section of the channel one observes a considerable variation (by a factor of 1.5-2) in the convective heat-transfer intensity. In addition, one can meet a nonequilibrium distribution of internal heat-generating sources in the cooled object. The result is that a temperature field forms in the object, and in some cases this leads to degradation of the properties of the object and even to its breakdown. For this reason the need arises to secure a specific temperature field in the cooled object.

We consider the one-dimensional steady-state heat-conduction equation for a rod with internal heat sources and with a given heat transfer from the surroundings through the lateral surface:

$$\frac{\partial^2 T(x)}{\partial x^2} - \frac{\alpha(x)\Pi}{\lambda(x)S(x)}[T(x) - T_{in}] + \frac{q(x)}{\lambda(x)S(x)} = 0. \qquad (1)$$

It was shown in [1] that in determining the local heat-transfer coefficient $\alpha(x)$ in the channel entrance section it is expedient to reference it to the difference between the local wall temperature T(x) and that of the coolant liquid T_{in} at the channel inlet.

We assume that we require to create a temperature field $T^*(x)$ which will guarantee normal operation of the element, and do so without altering its internal structure and the distribution of internal heat sources. The unique way to solve this problem is to provide the required distribution of convective heat-transfer coefficient over the surface of the element $\alpha(x)$, obtained from Eq. (1):

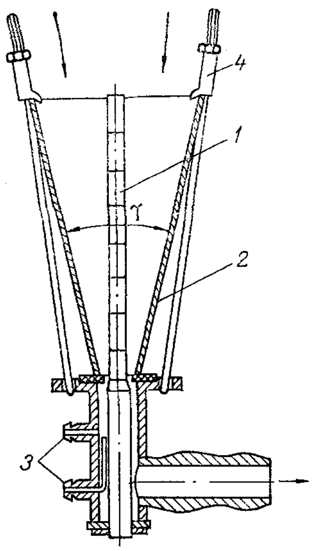

Fig. 1. Experimental arrangement.

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$$\alpha(x) = \left[\frac{\partial^2 T^*(x)}{\partial x^2} + \frac{q(x)}{\lambda(x) S(x)}\right] \frac{\lambda(x) S(x)}{\Pi [T^*(x) - T_{in}]}. \quad (2)$$

One can vary this coefficient by perturbing the liquid stream by setting up local turbulence generators, by developing heat-transfer surfaces by installing a fin, and also by using thermal insulation. We shall also examine a method of controlling the convective heat-transfer intensity, based on varying the liquid flow velocity in the channel, which is inversely proportional to the area in each channel section. The dimensions of the cross section of an annular channel depend on the longitudinal profile of its external wall.

In the literature one does not find investigations whose results would allow one to calculate the local convective heat-transfer coefficients in the entrance section of an annular channel when its cross section is changing, and we therefore conducted tests for which we used the facility depicted in Fig. 1. The main part is a cylindrical rod 1, located in the conical channel 2. We pump through air between the rod 1 and the cone 2, and measure the mass flow rate with the aid of the pitot tube 3, with an error not exceeding 5% at a confidence level of 0.95. The tube was previously calibrated with the help of standard Venturi tubes connected in series. As a result we established the dependence of the dynamic heat in the tube 3 on the volume flow rate of air.

The rod 1 is divided into 8 sections. Over the entire surface of each section is wound a Nichrome heater, and to the central part is attached the hot junction of a copper–constantan thermocouple. With a special supply unit one can measure and control the electrical power supplied to each heater in such a way that the temperatures of all the sections are the same. In this case the total electrical power W_i released in a section is carried away over the side surface by convection and radiation, and there is no interchange of heat between the sections.

To compute the experimental values of the heat-transfer coefficients we used the formula

$$\alpha_i = \frac{W_i - Q_i}{F(T_i - T_{in})}. \quad (3)$$

The values of α_i thus obtained are close to the values for x, corresponding to the midpoints of the sections. Analysis of the loss fluxes Q_i has shown that their magnitudes do not exceed 20% of W_i, and depend mainly on the radiant heat transfer between the surface of a section and the channel wall:

$$Q_i = 5.67 \cdot 10^{-8} \varepsilon_r F (T_i^4 - T_w^4). \quad (4)$$

The reduced emissivity ε_r is found from the formula [2]

$$\varepsilon_r = \left[\frac{1}{\varepsilon} + \frac{F}{F_c}\left(\frac{1}{\varepsilon_c} - 1\right)\right]^{-1}.$$

The values of emissivity of the surfaces of the rod and the cone were determined experimentally with the help of a type APIR-S pyrometer.

Calculation of the relative error in measuring the convective heat-transfer coefficients has shown that for the conditions of the experiments this does not exceed 12% at a confidence level of 0.95.

In the tests we used cones (Fig. 1) in which the angle γ was 0, 2, 6, 10, 14, 20, 30, 40, and 60°. The diameters of the lower ends of all the cones were the same. In the facility the cones were attached by clamps 4. The investigations were conducted at different air flow rates to achieve laminar and turbulent air flow regimes. The heat-transfer intensity in correlation form is given by the Nusselt number

$$\mathrm{Nu}_x = \frac{\alpha(x) x}{\lambda}. \quad (5)$$

The Reynolds number was determined from the formula

$$\mathrm{Re}_x = \frac{v(x) x}{\nu}. \quad (6)$$

The results of the experiments are shown in Fig. 2a and b. From a correlation of the test data one can propose the following theoretical relations: in turbulent air flow ($3 \cdot 10^4 < \mathrm{Re}_x < 15 \cdot 10^4$) and for angles $0° < \gamma < 40°$ we have

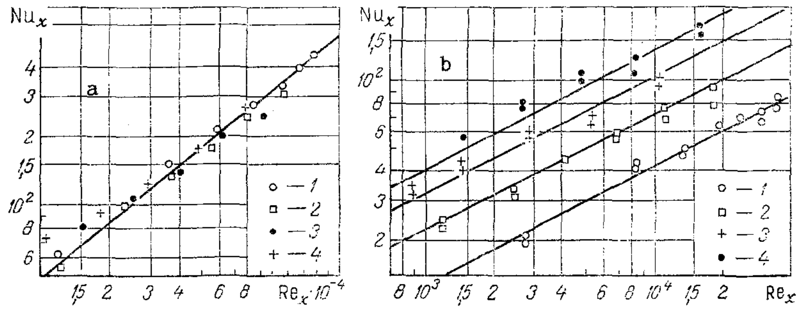

Fig. 2. Results of the experimental investigations of convective heat transfer with air in conical annular channels: a) heat transfer with turbulent flow of air (1, $\gamma = 0$; 2, 6; 3, 10; 4, 14°); b) with laminar flow (1, $\gamma = 0$; 2, 14; 3, 30; 4, 40°). γ is the angle between the channel generators.

$$\mathrm{Nu}_x = 0{,}0305\,\mathrm{Re}_x^{0.8}, \tag{7}$$

and in laminar flow for $40 < \mathrm{Re}_x < 2.5 \cdot 10^4$, $2 < D(x)/d < 15$ and angles $0° < \gamma < 40°$ we have

$$\mathrm{Nu}_x = 0.408\,\mathrm{Re}_x^{0.5}\left(1 - 2.8\frac{\partial D(x)}{\partial x}\right). \tag{8}$$

We also conducted tests with nozzles in which the angle γ was varied smoothly as a function of x. It turned out that Eqs. (7) and (8) can be used in such channels in the corresponding ranges with an error not exceeding 20%.

To calculate the channel profile that will satisfy condition (2), we substitute relations (5) and (6) into Eqs. (7) and (8), and solve them for $\alpha(x)$:

$$\alpha(x) = 0.305\,\lambda\left(\frac{v(x)}{\nu}\right)^{0.8} x^{-0.2}, \tag{9}$$

$$\alpha(x) = 0.408\lambda\left(\frac{v(x)}{x\nu}\right)^{0.5}\left(1 - 2.8\frac{\partial D(x)}{\partial x}\right). \tag{10}$$

The velocity v(x) can be determined from the volume flow rate of liquid and the local channel cross-sectional area M(x):

$$v(x) = G/M(x). \tag{11}$$

The cross-sectional area of the annular channel is determined from the formula

$$M(x) = \frac{\pi}{4}(D^2(x) - d^2). \tag{12}$$

Substituting Eqs. (11) and (12) into Eqs. (9) and (10), we obtain

$$\alpha(x) = 0.0305\lambda\left[\frac{4G}{\pi\nu(D^2(x) - d^2)}\right]^{0.8} x^{-0.2}, \tag{13}$$

$$\alpha(x) = 0.408\,\lambda\left[\frac{4G}{\pi\nu(D^2(x) - d^2)}\right]^{0.5}\left(1 - 2.8\frac{\partial D(x)}{\partial x}\right). \tag{14}$$

If we solve Eqs. (13) and (14) for D(x), we obtain a formula for calculating the channel profile that will satisfy condition (2).

Equation (13) can be solved analytically to give:

$$D(x) = \sqrt{\frac{0.0162}{\nu}\left[\frac{\lambda}{\alpha(x)}\right]^{1.25} x^{-0.25} + d^2}. \tag{15}$$

For a numerical solution of Eq. (14) we transform it to the form

$$Z(x) = \frac{1 - 2.8\dfrac{\partial D(x)}{\partial x}}{\sqrt{x(D^2(x) - d^2)}}, \tag{16}$$

where

$$Z(x) = \frac{\alpha(x)}{0{,}408\lambda}\left[\frac{\pi\nu}{4G}\right]^{0.5}. \tag{17}$$

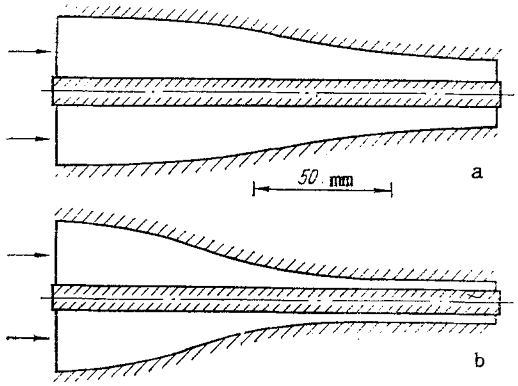

Fig. 3. Profiles of cooling channels calculated from Eq. (21) to achieve uniform rod surface temperature: a) with equilibrium distribution of heat sources; b) with a near-normal distribution of heat sources.

If the channel is divided along the coordinate x into sections of length 2δ, then ∂D(x)/∂x and Dx, for x corresponding to the midpoints of the sections, can be written approximately as:

$$\frac{\partial D(x)}{\partial x} \simeq \frac{D(x+\delta) - D(x-\delta)}{2\delta}, \tag{18}$$

$$D(x) \simeq \frac{D(x+\delta) + D(x-\delta)}{2}. \tag{19}$$

Substituting Eqs. (18) and (19) into Eq. (16), we obtain

$$Z(x) = \frac{1 - 2{,}8\,\dfrac{D(x+\delta) - D(x-\delta)}{2}}{\sqrt{x\left[\left(\dfrac{D(x+\delta) + D(x-\delta)}{2}\right)^2 - d^2\right]}}. \tag{20}$$

In Eq. (20) the quantities Z(x), δ, d, and x are given, while D(x − δ) are known from the previous section. In the first section the value of D(x − δ) may be overestimated in the system considered. Solving Eq. (20) for D(x + δ), we obtain

$$D(x+\delta) = \frac{-B + \sqrt{B^2 - 4AC}}{2A}, \tag{21}$$

where $A = Z^2(x)\delta^2 x - 7.84$; $B = D(x-\delta)(2A + 33.36) + 11.2\delta$; $C = D^2(x-\delta)A + 11.2\delta D(x-\delta) - 4Z^2(x)\delta^2 x d^2 - 4\delta^2$.

Subsequently, by finding the values of D(x + δ) from the first section, we can obtain the channel profile which will ensure the longitudinal temperature profile of the element being cooled. In the calculations we must take into account that the channel should not be expanded, since in that case Eqs. (7) and (8) are inapplicable. Using the method described we calculated and fabricated channels, shown in Fig. 3, for cooling elements. One channel (Fig. 3a) achieves a uniform temperature of 80°C along a cylindrical body of length 0.16 m and diameter 0.01 m. The internal heat sources are uniformly distributed along the body. The air flow rate was $1.37 \cdot 10^{-3}$ m^3/sec. In the calculations we took into account that a part of the heat flux from the body is carried away by radiation to the channel surface. To achieve the required surface temperature the convective heat-transfer coefficient, as calculated from Eq. (2), must be equal to 20.3 W/m^2·K. The inlet diameter was taken to be 0.052 m. In the central part of the channel the angle γ = 13.5°. It was pointed out in [3] that a nozzle with this opening angle has the greatest flow rate coefficient.

Figure 3b shows a channel to achieve a uniform temperature field for a cylindrical body at 80°C. The distribution of internal heat sources in the body is close to a normal distribution, with a maximum at the midpoint of its longitudinal axis, and varying from 32 W/m at the ends of the body to 62 W/m in the middle.

The investigations carried out with these channels on the facility described in Fig. 1 have shown that the temperature drops along the body did not exceed 1-2°K. If we use for these bodies a channel of constant section with an outer diameter of 0.023 m, then the temperature drops along the length are not less than 30°K for the same flow rate of cooling liquid.

On the basis of the above we can recommend this method for obtaining thermal conditions for certain elements in the instrumentaion field.

NOTATION

x, coordinate, m; T, T^*, T_{in}, temperatures of the element surface (actual and assigned) and of air at the channel inlet, °K; α(x), coefficient of convective heat transfer, $W/m^2 \cdot K$; λ(x), thermal conductivity of the element material, W/m·K; S(x) and Π, area of cross section and perimeter of the element being cooled, m^2, m; q(x), specific power of the heat sources in the element, W/m; W_i, electrical power supplied to section i, W; O_i, loss flux, W; F, section area, m^2; F_c, area of the internal cone surface, m^2; ε and ε_c, emissivities of the rod and cone surfaces; ε_r, reduced emissivity; D(x), diameter of the internal channel wall, m; d, diameter of the element being cooled, m; λ, thermal conductivity of the liquid, W/m·K; ν, viscosity of the liquid, m^2/sec; M(x), flow area of the channel, m^2; T_i, T_w, temperatures of section i of rod 1 and of the wall of channel 2 (Fig. 1), respectively, °K.

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HEAT TRANSFER IN TURBULENT FREE CONVECTION AROUND A HORIZONTAL NONISOTHERMAL CYLINDER

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The heat transfer of a horizontal cylinder in turbulent free convection and for a quadratic law of temperature variation on its surface is investigated numerically: $10^9 \le Ra \le 10^{13}$, Pr = 0.71.

Heat transfer with free convection at horizontal circular cylinders has been studied repeatedly both experimentally and theoretically [1-5]. Development of a turbulent flow regime is considered below for nonisothermal boundary conditions that can hold near the surface of powerful thermal power plant elements [6].

The mathematical model of the process is the Reynolds equation in the Boussinesq approximation which reduces to a system of three differential equations of identical structure [5] after going over to dimensionless form and using the new independent variable ξ = ln R.

$$\exp(-2\xi)\left\{\frac{\partial}{\partial\varphi}\left(\gamma\Phi\frac{\partial\psi}{\partial\xi}-\zeta\frac{\partial(\eta\Phi)}{\partial\varphi}\right)-\frac{\partial}{\partial\xi}\left(\gamma\Phi\frac{\partial\psi}{\partial\varphi}\right)+\zeta\frac{\partial(\eta\Phi)}{\partial\xi}\right)\right\}=S, \quad (1)$$

where the average axial component of vorticity, the stream function, and temperature are considered, respectively, as the desired function Φ. The specific form of the coefficients of (1) is represented in Table 1. The following scales, r_0, a/r_0, $t_m - t_f$, are chosen for the coordinate, the average velocity vector component, and the temperature.

The problem was solved for the boundary conditions

$$\xi=0\quad \partial\psi/\partial\xi=\psi=0,\ T=T(\varphi);\qquad \xi=5\quad T=\Omega=\psi=0;\qquad \varphi=0,\pi\quad \psi=\Omega=\partial T/\partial\varphi=0.$$

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